Distance (d) equals the product of the rate of speed (r) and the time (t). This relationship is shown below in three forms:

$$d = r \cdot t \qquad r = \frac{d}{t} \qquad t = \frac{d}{r}$$

It is important that the units of measure are consistent.

Example 1

Find the rate of speed of a passenger car if the distance traveled is 572 miles and the time elapsed is 11 hours.

$$572 \text{ miles} = r \cdot 11 \text{ hours} \implies \frac{572 \text{ miles}}{11 \text{ hours}} = r \implies 52 \text{ miles/hour} = \text{rate}$$

Example 2

Find the distance traveled by a train at 135 miles per hour for 40 minutes.

The units of time are not the same so we need to change 40 minutes into hours. $\frac{40}{60} = \frac{2}{3}$ hour.

$$d = (135 \text{ miles/hour})(\tfrac{2}{3} \text{ hour}) \implies d = 90 \text{ miles}$$

Example 3

The Central Middle School hamster race is fast approaching. Fred said that his hamster traveled 60 feet in 90 seconds and Wilma said she timed for one minute and her hamster traveled 12 yards. Which hamster has the fastest rate?

$\text{rate} = \frac{\text{distance}}{\text{time}}$ but all the measurements need to be in the same units. In this example, we use feet and minutes.

Fred's hamster: $\text{rate} = \frac{60 \text{ feet}}{1.5 \text{ minutes}} \implies \text{rate} = 40$ feet/minute

Wilma's hamster: $\text{rate} = \frac{36 \text{ feet}}{1 \text{ minute}} \implies \text{rate} = 36$ feet/minute

Fred's hamster is faster.

Problems

Solve the following problems.

1. Find the time if the distance is 157.5 miles and the speed is 63 mph.

2. Find the distance if the speed is 67 mph and the time is 3.5 hours.

3. Find the rate if the distance is 247 miles and the time is 3.8 hours.

4. Find the distance if the speed is 60 mph and the time is 1 hour and 45 minutes.

5. Find the rate in mph if the distance is 3.5 miles and the time is 20 minutes.

6. Find the time in minutes if the distance is 2 miles and the rate is 30 mph.

7. Which rate is faster? A: 60 feet in 90 seconds or B: 60 inches in 5 seconds

8. Which distance is longer? A: 4 feet/second for a minute or B: 3 inches/min for an hour

9. Which time is shorter? A: 4 miles at 60 mph or B: 6 miles at 80 mph

Answers

| 1. | 2.5 hr | 2. | 234.5 mi | 3. | 65 mph | 4. | 105 miles | 5. | 10.5 mph |
| 6. | 4 min | 7. | B | 8. | A | 9. | A | | |

Students used scale factors (multipliers) to enlarge and reduce figures as well as increase and decrease quantities. All of the original quantities or lengths were multiplied by the scale factor to get the new quantities or lengths. To reverse this process and scale from the new situation back to the original, we divide by the scale factor. Division by a scale factor is the same as multiplying by a reciprocal. This same concept is useful in solving equations with fractional coefficients. To remove a fractional coefficient you may divide each term in the equation by the coefficient or multiply each term by the reciprocal of the coefficient. Recall that a reciprocal is the multiplicative inverse of a number, that is, the product of the two numbers is 1. For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, $\frac{1}{2}$ is $\frac{2}{1}$, and 5 is $\frac{1}{5}$.

Scaling may also be used with percentage problems where a quantity is increased or decreased by a certain percent. Scaling by a factor of 1 does not change the quantity. Increasing by a certain percent may be found by multiplying by (1 + the percent) and decreasing by a certain percent may be found by multiplying by (1 – the percent).

For additional information, see the Math Notes box in Lesson 7.1.4 of the *Core Connections, Course 2* text.

Example 1

The large triangle at right was reduced by a scale factor of $\frac{2}{5}$ to create a similar triangle. If the side labeled x now has a length of 80' in the new figure, what was the original length?
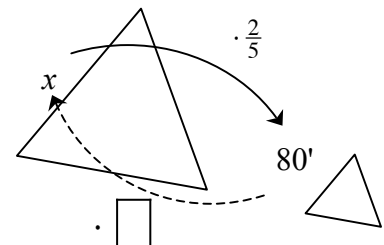
To undo the reduction, multiply 80' by the reciprocal of $\frac{2}{5}$, namely $\frac{5}{2}$, or divide 80' by $\frac{2}{5}$.

$80' \div \frac{2}{5}$ is the same as $80' \cdot \frac{5}{2}$, so $x = 200'$.

Example 2

Solve: $\frac{2}{3}x = 12$

Method 1: Use division and a Giant One

$$\frac{2}{3}x = 12$$

$$\frac{\frac{2}{3}x}{\frac{2}{3}} = \frac{12}{\frac{2}{3}}$$

$$x = \frac{12}{\frac{2}{3}} = 12 \div \frac{2}{3} = \frac{36}{3} \div \frac{2}{3} = \frac{36}{2} = 18$$

Method 2: Use reciprocals

$$\frac{2}{3}x = 12$$

$$\frac{3}{2}\left(\frac{2}{3}x\right) = \frac{3}{2}(12)$$

$$x = 18$$

Example 3

Samantha wants to leave a 15% tip on her lunch bill of $12.50. What scale factor should be used and how much money should she leave?

Since tipping increases the total, the scale factor is $(1 + 15\%) = 1.15$.
She should leave $(1.15)(12.50) = \$14.38$ or about $14.50.

Example 4

Carlos sees that all DVDs are on sales at 40% off. If the regular price of a DVD is $24.95, what is the scale factor and how much is the sale price?

If items are reduced 40%, the scale factor is $(1 - 40\%) = 0.60$.
The sale price is $(0.60)(24.95) = \$14.97$.

Problems

1. A rectangle was enlarged by a scale factor of $\frac{5}{2}$ and the new width is 40 cm. What was the original width?

2. A side of a triangle was reduced by a scale factor of $\frac{2}{3}$. If the new side is now 18 inches, what was the original side?

3. The scale factor used to create the design for a backyard is 2 inches for every 75 feet ($\frac{2}{75}$). If on the design, the fire pit is 6 inches away from the house, how far from the house, in feet, should the fire pit be dug?

4. After a very successful year, Cheap-Rentals raised salaries by a scale factor of $\frac{11}{10}$. If Luan now makes $14.30 per hour, what did she earn before?

5. Solve: $\frac{3}{4}x = 60$ 6. Solve: $\frac{2}{5}x = 42$

7. Solve: $\frac{3}{5}y = 40$ 8. Solve: $-\frac{8}{3}m = 6$

9. What is the total cost of a $39.50 family dinner after you add a 20% tip?

10. If the current cost to attend Magicland Park is now $29.50 per person, what will be the cost after a 8% increase?

11. Winter coats are on clearance at 60% off. If the regular price is $79, what is the sale price?

12. The company president has offered to reduce his salary 10% to cut expenses. If she now earns $175,000, what will be her new salary?

Answers

1. 16 cm

2. 27 inches

3. $18\frac{3}{4}$ feet

4. $13.00

5. 80

6. 105

7. $66\frac{2}{3}$

8. $-2\frac{1}{4}$

9. $47.40

10. $31.86

11. $31.60

12. $157,500

EQUATIONS WITH FRACTIONAL COEFFICIENTS

Students used scale factors (multipliers) to enlarge and reduce figures as well as increase and decrease quantities. All of the original quantities or lengths were multiplied by the scale factor to get the new quantities or lengths. To reverse this process and scale from the new situation back to the original, we divide by the scale factor. Division by a scale factor is the same as multiplying by a reciprocal. This same concept is useful in solving one-step equations with fractional coefficients. To remove a fractional coefficient you may divide each term in the equation by the coefficient or multiply each term by the reciprocal of the coefficient.

To remove fractions in more complicated equations students use "Fraction Busters." Multiplying all of the terms of an equation by the common denominator will remove all of the fractions from the equation. Then the equation can be solved in the usual way.

For additional information, see the Math Notes box in Lesson 7.1.6 of the *Core Connections, Course 2* text.

Example of a One-Step Equation

Solve: $\frac{2}{3}x = 12$

Method 1: Use division and common denominators

$$\frac{2}{3}x = 12$$

$$\frac{\frac{2}{3}x}{\frac{2}{3}} = \frac{12}{\frac{2}{3}}$$

$$x = \frac{12}{\frac{2}{3}} = 12 \div \frac{2}{3} = \frac{36}{3} \div \frac{2}{3} = \frac{36}{2} = 18$$

Method 2: Use reciprocals

$$\frac{2}{3}x = 12$$

$$\frac{3}{2}\left(\frac{2}{3}x\right) = \frac{3}{2}(12)$$

$$x = 18$$

Example of Fraction Busters

Solve: $\frac{x}{2} + \frac{x}{5} = 6$

Multiplying by 10 (the common denominator) will eliminate the fractions.

$$10(\tfrac{x}{2} + \tfrac{x}{5}) = 10(6)$$

$$10(\tfrac{x}{2}) + 10(\tfrac{x}{5}) = 10(6)$$

$$5x + 2x = 60$$

$$7x = 60 \Rightarrow x = \tfrac{60}{7} \approx 8.57$$

Problems

Solve each equation.

1. $\frac{3}{4}x = 60$ 2. $\frac{2}{5}x = 42$

3. $\frac{3}{5}y = 40$ 4. $-\frac{8}{3}m = 6$

5. $\frac{3x+1}{2} = 5$ 6. $\frac{x}{3} - \frac{x}{5} = 3$

7. $\frac{y+7}{3} = \frac{y}{5}$ 8. $\frac{m}{3} - \frac{2m}{5} = \frac{1}{5}$

9. $-\frac{3}{5}x = \frac{2}{3}$ 10. $\frac{x}{2} + \frac{x-3}{5} = 3$

11. $\frac{1}{3}x + \frac{1}{4}x = 4$ 12. $\frac{2x}{5} + \frac{x-1}{3} = 4$

Answers

1. $x = 80$ 2. $x = 105$ 3. $y = 66\frac{2}{3}$ 4. $m = -\frac{9}{4}$

5. $y = 3$ 6. $x = 22.5$ 7. $y = -17\frac{1}{2}$ 8. $m = -3$

9. $x = -\frac{10}{9}$ 10. $x = \frac{36}{7}$ 11. $x = \frac{48}{7}$ 12. $x = \frac{65}{11}$

A percent increase is the amount that a quantity has increased based on a percent of the original amount. A percent decrease is the amount that a quantity has decreased based on a percent of the original amount. An equation that represents either situation is:

amount of increase or decrease = (% change)(original amount)

For additional information see the Math Notes box in Lesson 7.1.1 of the *Core Connections, Course 2* text.

Example 1

A town's population grew from 1879 to 7426 over five years. What was the percent increase in the population?

- Subtract to find the change:

$$7426 - 1879 = 5547$$

- Put the known numbers in the equation:

$$5547 = (x)(1879)$$

- The scale factor becomes x, the unknown:

$$\frac{5547}{1879} = x$$

- Divide: $x = \frac{5547}{1879} \approx 2.952$

- Change to percent: $x = 295.2\%$

The population increased by 295.2%.

Example 2

A sumo wrestler retired from sumo wrestling and went on a diet. When he retired he weighed 385 pounds. After two years he weighed 238 pounds. What was the percent decrease in his weight?

- Subtract to find the change:

$$385 - 238 = 147$$

- Put the known numbers in the equation:

$$147 = (x)(385)$$

- The scale factor becomes x, the unknown:

$$\frac{147}{385} = x$$

- Divide: $x = \frac{147}{385} \approx 0.382$

- Change to percent: $x \approx 38.2\%$

His weight decreased by about 38.2 %.

Problems

Solve the following problems.

1. Forty years ago gasoline cost $0.30 per gallon on average. Ten years ago gasoline averaged about $1.50 per gallon. What is the percent increase in the cost of gasoline?

2. When Spencer was 5, he was 28 inches tall. Today he is 5 feet 3 inches tall. What is the percent increase in Spencer's height?

3. The cars of the early 1900s cost $500. Today a new car costs an average of $27,000. What is the percent increase of the cost of an automobile?

4. The population of the U.S. at the first census in 1790 was 3,929 people. By 2000 the population had increased to 284,000,000! What is the percent increase in the population?

5. In 2000 the rate for a first class U.S. postage stamp increased to $0.34. This represents a $0.31 increase since 1917. What is the percent increase in cost since 1917?

6. In 1906 Americans consumed an average of 26.85 gallons of whole milk per year. By 1998 the average consumption was 8.32 gallons. What is the percent decrease in consumption of whole milk?

7. In 1984 there were 125 students for each computer in U.S. public schools. By 1998 there were 6.1 students for each computer. What is the percent decrease in the ratio of students to computers?

8. Sara bought a dress on sale for $30. She saved 45%. What was the original cost?

9. Pat was shopping and found a jacket with the original price of $120 on sale for $9.99. What was the percent decrease in the cost?

10. The price of a pair of pants decreased from $49.99 to $19.95. What was the percent decrease in the price?

Answers

1.	400%	2.	125%	3.	5300%	4.	7,228,202.4%
5.	91.2%	6.	69.01%	7.	95.12%	8.	$55
9.	91.7%	10.	60.1%				

In Course 2 students are introduced to simple interest, the interest is paid only on the original amount invested. The formula for simple interest is: $I = Prt$ and the total amount including interest would be: $A = P + I$.

For additional information, see the Math Notes box in Lesson 7.1.8 of the *Core Connections, Course 2* text.

Example

Wayne earns 5.3% simple interest for 5 years on $3000. How much interest does he earn and what is the total amount in the account?

Put the numbers in the formula $I = Prt$. $I = 3000(5.3\%)5$

Change the percent to a decimal. $= 3000(0.053)5$

Multiply. $= 795$ Wayne would earn $795 interest.

Add principal and interest. $3000 + $795 = $3795 in the account

Problems

Solve the following problems.

1. Tong loaned Jody $50 for a month. He charged 5% simple interest for the month. How much did Jody have to pay Tong?

2. Jessica's grandparents gave her $2000 for college to put in a savings account until she starts college in four years. Her grandparents agreed to pay her an additional 7.5% simple interest on the $2000 for every year. How much extra money will her grandparents give her at the end of four years?

3. David read an ad offering $8\frac{3}{4}\%$ simple interest on accounts over $500 left for a minimum of 5 years. He has $500 and thinks this sounds like a great deal. How much money will he earn in the 5 years?

4. Javier's parents set an amount of money aside when he was born. They earned 4.5% simple interest on that money each year. When Javier was 15, the account had a total of $1012.50 interest paid on it. How much did Javier's parents set aside when he was born?

5. Kristina received $125 for her birthday. Her parents offered to pay her 3.5% simple interest per year if she would save it for at least one year. How much interest could Kristina earn?

Answers

1. $I = 50(0.05)1 = \$2.50$; Jody paid back $52.50.

2. $I = 2000(0.075)4 = \$600$

3. $I = \$500(0.0875)5 = \$218.75$

4. $\$1012.50 = x(0.045)15$; $x = \$1500$

5. $I = 125(0.035)1 = \$4.38$

Students represent distributions of single-variable data numerical data using dot plots, stem-and-leaf plots, box plots, and histograms. They represent categorical one-variable data on bar graphs.   Each representation communicates information in a slightly different way.
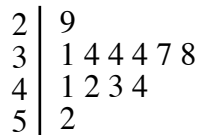
STEM-AND-LEAF-PLOTS

A stem-and-leaf plot is a way to display data that shows the individual values from a set of data and how the values are distributed. The "stem" part on the graph represents all of the digits except the last one. The "leaf" part of the graph represents the last digit of each number.

Read more about stem-and-leaf plots, and how they compare to dot plots and histograms, in the Math Notes box in Lesson 7.1.1 of the *Core Connections, Course 2* text.
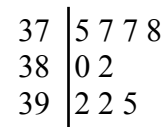
Example 1

Make a stem-and-leaf plot of this set of data: 34, 31, 37, 44, 38, 29, 34, 42, 43, 34, 52, and 41.

```
2 | 9
3 | 1 4 4 4 7 8
4 | 1 2 3 4
5 | 2
```

Example 2

Make a stem-and-leaf plot of this set of data: 392, 382, 380, 392, 378, 375, 395, 377, and 377.

```
37 | 5 7 7 8
38 | 0 2
39 | 2 2 5
```

Problems

Make a stem-and-leaf plot of each set of data.

1.   29, 28, 34, 30, 33, 26, 18, and 34.

2.   25, 34, 27, 25, 19, 31, 42, and 30.

3.   80, 89, 79, 84, 95, 79, 89, 67, 82, 76, 92, 89, 81, and 123.

4.   116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.

Answers

1.

1	8
2	6 8 9
3	0 3 4 4

2.

1	9
2	5 5 7
3	0 1 4 5
4	2

3.

6	7
7	6 9 9
8	0 1 2 4 9 9 9
9	2 5
10	
11	
12	3

4.

9	1
10	0 1 1 3 4 5 7 8 9
11	1 3 3 6 8